Log Models

Log Functions Day 7:
- (2.14) Logarithmic Models

The graph and table below give data on the salary of clergy members across the US and the size of their congregations.

Congregation size	75	120	250	300	450	650	900	1500	2500	5000
Salary	42	58	70	79	88	95	110	125	138	175

What type of regression should we perform to model this data?

Log Models

Log Functions Day 7:

- (2.14) Logarithmic Models

Homework: Logarithmic Modelling + Calculator Practice

Log Models

Log Models are of the form $f(x) = a + b \ln x$ or $a + b \log x$

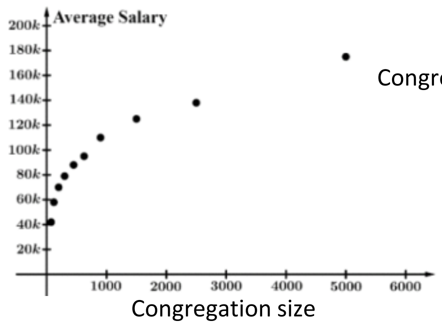
(why does the log base not really matter?)

We use log models when input seems to change proportionally over equal-length output-value intervals

x	5	10	20	40	80	160
y	-2	-1	0	1	2	3

(stolen from Bryan Passwater)

The graph and table below give data on the salary of clergy members across the US and the size of their congregations.



Congregation size	75	120	250	300	450	650	900	1500	2500	5000
Salary	42	58	70	79	88	95	110	125	138	175

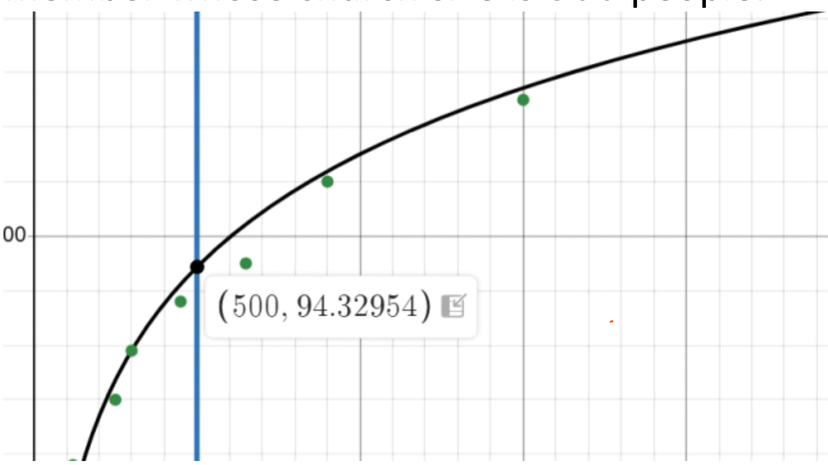
(stolen from Bryan Passwater)

Church Size	75	120	250	300	450	650	900	1500	2500	5000
Avg. Salary (in 1000's of dollars)	42	58	70	79	88	95	110	125	138	175

A) Use the regression capabilities of your calculator to find a logarithmic function that models the data.

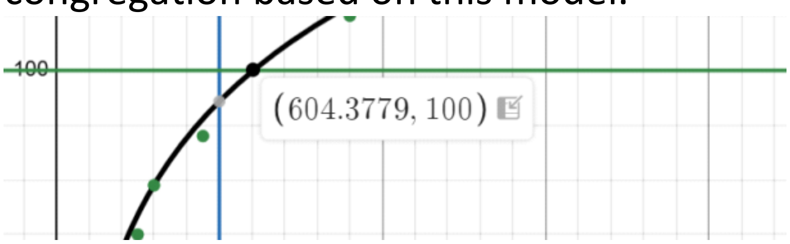
$$y = -91.54204 + 29.90882 \cdot \ln(x)$$

B) Using the model, predict the annual salary, in thousands of dollars, for a clergy member whose church size is 500 people.



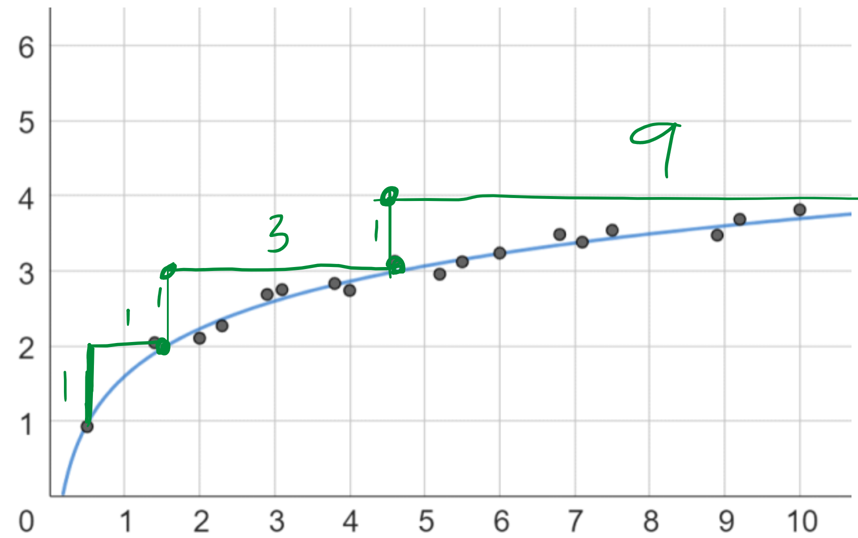
94.330 thousand

C) If a clergy member makes a $100,000 annual salary, predict the size of their congregation based on this model.



604.378 people

18 data points are plotted below. They are modeled well by a function of the form $f(x) = a + \log_b x$. What base logarithm would be most appropriate?



(A) $b = 2$

(B) $b = 3$

(C) $b = 4$

(D) $b = 5$

x	-2	-1	0	1	2	3
y	4	12	36	108	324	972

$f(x) = a(b)^x$

The most common method to describe sound intensity level is in decibels (dB). The intensity level β (in decibels) of a sound having intensity I (in watts per meter squared) is modeled by a function of the following form.

$$\beta(I) = a + b\log(I)$$

Noisy traffic has intensity 1×10^{-5} watts per meter squared and that corresponds with an intensity level of 70 dB.

A loud concert has intensity 1 watt per meter squared and that corresponds with an intensity level of 120 dB.

A) Using a calculator/Desmos/your impressive brains, find the value of a and b

B) The human eardrum can burst if exposed to 1×10^{4} watts per meter squared. According to the model found above, what is the predicted number of decibels that would burst the human eardrum?

The function f is of the form $f(x) = a + b\log(x - 1)$

$f(6) = 7$ and $f(9) = 11$

A) Find a and b

B) Find $f(10)$

C) Find $f^{-1}(12)$

The function f is of the form $f(x) = a + b\log(x-1)$

$f(6) = 7$ and $f(9) = 11$

D) Find the average rate of change of f on the interval $6 \le x \le 9$

E) Use the average rate of change from part (D) to estimate $f(11)$

F) Using the shape of the graph of $f(x)$ explain how one knows that the answer to (E) overestimates the actual value of $f(11)$.

Without looking at your notes to figure out the wording, try to justify why you believe the values in the following table would be modelled well by a logarithmic function.

x	100	50	25	12.5	6.25	3.125
$g(x)$	-4	0	4	8	12	16

FRQ Practice

As a reminder, the structure of FRQ2 is:

- Context that includes two to four points.
- The type of equation we'd like to use to model the context.

- A:
  - Part i: Set up a system of equations to solve for a, b, c (no work necessary)
  - Part ii: Solve for a, b, c (no work necessary)
- B:
  - Find an average rate of change
  - Apply the average rate of change (often, use average rate of change to estimate)
  - Reason using the average rate of change (often, use concavity to explain why estimate based on AROC is greater/less than value from model)
- C: Explain limitation of the model (domain restriction, range restrictions, other)

Part A

Part i: No work necessary

Day 8 Practice FRQ 2

Part ii: No work necessary

Day 8 FRQ #2 Practice

t	0	2	5
Thousands of Pounds	20	35	55

At the start of 2015, Ira opened a new business turning recycled glass into sand. At the end of each year, he reports to his social media followers how many total pounds of glass he has recycled since the business opened. At the end of 2015 ($t = 0$), he reports that he recycled 20 thousand pounds of glass. At the end of 2017 ($t = 2$), he reports that he recycled 35 thousand pounds of glass since the business began. At the end of 2020 ($t = 5$), he reports that he recycled 55 thousand pounds of glass since the business began.

The amount of glass that Ira's business recycled can be modeled by the quadratic function $R(t) = at^2 + bt + c$, where $R(t)$ is the total amount recycled, in thousands of pounds, since the business began, and t is years since the business opened.

(A) (i) Use the given data to write three equations that can be used to find the values for the constants a, b, and c in the expression for $R(t)$.
(ii) Find the values for a, b, and c as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the total recycled amount, in thousands of pounds per year, from $t = 2$ to $t = 5$. Express your answer as a decimal approximation. Show the computations that lead to your answer.
(ii) Use the average rate of change found in part B(i) to estimate the total amount recycled at the end of 2016 ($t = 1$). Show the work that leads to your answer.
(iii) Let A_t represent the estimate for the total amount recycled, in thousands of pounds, using the average rate of change found in part B(i). For example, in part B(ii), A_1 was calculated. Explain why, in general, $A_t > R(t)$ for all t where $0 < t < 2$. Your explanation should include a reference to the graph of R and its relationship to A_t.

(C) The quadratic function model R has exactly one absolute minimum or absolute maximum. That minimum or maximum can be used to determine a domain restriction for R. Based on the context of the problem, explain how that minimum or maximum can be used to determine a boundary for the domain of R.

$20 = a(0)^2 + b(0) + c$

$35 = a(2)^2 + b(2) + c$

$55 = a(5)^2 + b(5) + c$

$a = -0.166667$ $b = 7.83333$

$c = 20$

$a = -0.167$

$b = 7.833$

$c = 20$

Part B

- The work for average rate of change should look like a subtraction problem divided by a subtraction problem
- Explaining why estimates based on average rate of change over/under estimate values of a model should include:
  - Quick sketch with t-values and graphs labelled
  - Talk about concavity of the model
  - Refer to the estimates based on AROC as outputs of a secant line
  - t-values of secant line
  - t-values the problem is about
  - Which graph is on top

t	0	2	5
Thousands of Pounds	20	35	55

(B) (i) Use the given data to find the average rate of change of the total recycled amount, in thousands of pounds per year, from $t = 2$ to $t = 5$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

(ii) Use the average rate of change found in part B(i) to estimate the total amount recycled at the end of 2016 ($t = 1$). Show the work that leads to your answer.

(iii) Let A_t represent the estimate for the total amount recycled, in thousands of pounds, using the average rate of change found in part B(i). For example, in part B(ii), A_1 was calculated. Explain why, in general, $A_t > R(t)$ for all t where $0 < t < 2$. Your explanation should include a reference to the graph of R and its relationship to A_t.

B(i) $\dfrac{R(5) - R(2)}{5-2} = \dfrac{55 - 35}{5 - 2} = \dfrac{20}{3} = 6.667$

B(ii)

$(1, R(1))$

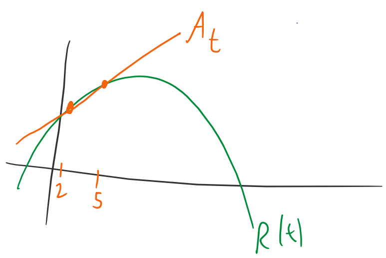
$(2, 35)$

$\dfrac{R(1) - 35}{1-2} \approx 6.667$

$\dfrac{R(1) - 35}{-1} \approx 6.667$

$R(1) - 35 \approx -6.667$

$R(1) \approx 28.333$



$R(t)$ is concave down

A_t is the output of the secant line passing through R at $t = 2$ and $t = 5$

Since R is concave down, the secant line is above the graph of R for values of t in $0 < t < 2$

Part C

For domain/range restrictions, write something like:

For ____________ input/output values,
the model ____________,
but in reality _________________.
So we restrict __________________.

For t values greater than the t value of the absolute maximum, the model decreases. But in reality, total amount recycled cannot decrease. So, we restrict the domain to $t <$ the t value of the maximum.